School of Mathematics and Statistics

MT2505 Abstract Algebra

Problem Sheet II: Greatest common divisors; Euclidean Algorithm

1. Let a, b, c, d, x and y be integers.
   (a) If $a \mid b$ and $b \mid c$, show that $a \mid c$.
   (b) If $a \mid b$ and $a \mid c$, show that $a \mid (bx + cy)$.
   (c) If $a \mid b$ and $c \mid d$, show that $ac \mid bd$.

2. Let a, b, q and r be integers with $a \neq 0$ such that $a = qb + r$. Show that

$$\gcd(a, b) = \gcd(b, r).$$

   [Hint: First show that $d = \gcd(a, b)$ divides r and then use the definition of the greatest common divisor.]

3. (a) Find the greatest common divisor of 48 and 174. Find the Bézout coefficients u and v to express this greatest common divisor in the form $48u + 174v$.
   (b) Find the greatest common divisor of 196 and 238. Find the Bézout coefficients u and v to express this greatest common divisor in the form $196u + 238v$.
   (c) Find the greatest common divisor of 2619 and 783. Find the Bézout coefficients u and v to express this greatest common divisor in the form $2619u + 783v$.

4. Let a and b be integers at least one of which is non-zero. Define $d = \gcd(a, b)$, the greatest common divisor of a and b.

   Show that

$$\gcd(a/d,\, b/d) = 1.$$

   [Hint: If x divides a/d, show that xd divides a. How does this help you?]